Multiplication Games:

How We Made and Used Them

Teachers introduce multiplication in kindergarten and the first two grades in the form of word problems such as the following: "I want to give 2 pieces of chocolate to each person in my family. There are 5 people in my family. How many pieces of chocolate do I need?" Children usually use repeated addition to solve such problems, as Carpenter et al. (1993) and Kamii (2000) describe. By third grade, however, many children begin to use multiplication as they become capable of multiplicative thinking (Clark and Kamii 1996).

Some educators think that teachers should teach for understanding of multiplication rather than for speed. This probably is a reaction to teachers' common practice of making systematic use of timed tests without any reflection, for example, about the relationship between the table of 2s and the table of 4s. In our opinion, children should have an understanding of multiplication and should develop speed. With our advanced third graders in a Title I school, therefore, we have been using games instead of worksheets or timed tests after the children have developed the logic of multiplication. The results have been encouraging. Toward the end of the school year, when the children had played multiplication games for several months, we gave a summative-evaluation test consisting of one hundred multiplication problems to finish in ten minutes. Every child in the class except one (who made two errors) wrote one hundred correct answers within the time limit. This article describes some of the games we used, how we modified commercially made games, and what we learned by using them.

Seven games are described under three headings: a game involving one multiplication table at a time, games involving many multiplication tables and small but increasing factors, and games requiring speed.

A Game Involving One Table at a Time

Rio is a game that is best played by three children. If there are four players, turns come less frequently, and children will be less active mentally. Rio uses ten tiles or squares made with cardboard, fifteen transparent chips (five each of three different colors), and a ten-sided number cube showing the numbers 1–10. For the table of 4s, for example, we wrote the ten products (4, 8, 12, 16, 20, 24, 28, 32, 36, and 40) on the tiles. These tiles are scattered in the middle of the table, and each player takes five chips of the same color.

The first player rolls the number cube, and if a

By Constance Kamii and Catherine Anderson


Constance Kamii teaches at the University of Alabama at Birmingham. Her interest is in figuring out how to improve mathematics education for young children by using Piaget's theory. Catherine Anderson holds a master's degree in education and has taught for more than fifteen years. Twelve of those years have been at Rio Vista Elementary School, a Title I, NCLB, under-performing school. Her focus areas are the development of subtraction and multiplicative thinking.

Children practice a multiplication game.

Figure 1

Easy products and increasingly greater factors

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5				
2	2	4	6	8	10				
3	3	6	9	12	15				
4	4	8	12	16	20				
5	5	10	15	20	25				
6									
7									
8									
9									

5 comes up, for example, he or she puts a chip on the tile marked "20" for 5 × 4. The second player then rolls the number cube, and if an 8 comes up, he or she puts a chip on 32 for 8 × 4. If the third player rolls a 5, the tile marked "20" already has a chip on it, so the player must take it. The third player now has six chips and the first player has four. Play continues in this way, and the person who plays all his or her chips first is the winner.

This is a good introductory game, and most third graders begin by using repeated addition rather than multiplication. As they continue to play Rio, finding products when multiplying by 2 and 10 becomes easy. The next products that they master are multiples of 5 and 3. Multiplying by 6, 7, 8, and 9 is much more difficult. The next category of games is more appropriate after this introduction to all the tables.

Games Involving Many Tables and Small but Increasing Factors

Figure 1 shows easy products of factors up to 5. When children know these products very well, teachers can introduce factors up to 6, 7, and so on.

Examples of games in this category are Salute, Four-in-a-Row, and Winning Touch.

Salute

In Salute, three players use part of a deck of playing cards. At first, we use the twenty cards A–5 and remove all the others (6–K). Ace counts as one. Later, we use the twenty-four cards A–6, then A–7 (twenty-eight cards), and so on.

The dealer holds the twenty cards A–5—or forty cards if two decks are used—and hands a card to each of the two players without letting anyone see the numbers on them. The two players then simultaneously say "Salute!" as they each hold a card to their foreheads in such a way that they can see the opponent's card but not their own. The dealer, who can see both cards, announces the product of the two numbers, and each player tries to figure out the factor on his or her card. The player who announces the correct factor first wins both cards. The winner of the game is the player who has more cards at the end. (We decided that the dealer should hold the deck because when the cards were dealt, the players confused their "winnings" with the cards they had yet to use.)

When this game becomes too easy, children can use cards up to 6, 7, and so on, as stated earlier.

Four-in-a-Row

This is a two-player game that uses a board such as the one in **figure 2a,** eighteen transparent chips of one color, eighteen transparent chips of another color, and two paper clips. Each player takes eighteen chips of the same color to begin the game. The first player puts the two paper clips on any two numbers at the bottom outside the square, such as the 4 and the 5. The same player then multiplies these numbers and puts one of his or her eighteen chips on any 20 because $4 \times 5 = 20$.

The second player moves one of the two paper clips that are now on the 4 and the 5. If the second player moves one of them from 4 to 3, this person can place one of his or her eighteen chips on any 15 because $3 \times 5 = 15$. On every subsequent turn, a player must move one of the two paper clips to a different number. Two paper clips can be placed on the same number, to make 5×5, for example. The person who is first to make a line of four chips of the same color, vertically, horizontally, or diagonally, is the winner.

The reader may have seen a Four-in-a-Row board such as the one in **figure 2b.** This board is not ideal because some children use only the factors up to 4 or 5. The board in **figure 2a** is better because it does not involve easy factors such as 1 and 2 and more difficult factors such as 7, 8, and 9. The range of factors from 3 to 6 is more appropriate at the beginning because it focuses children's efforts on a few combinations at the correct level of difficulty. When the board in **figure 2a** becomes too easy, teachers can introduce factors 3–7 and a new board made with appropriate products.

We randomly scattered the numbers on the board in **figure 2a** and chose them in the following way. The board includes ten combinations of factors 3–6 because there are four combinations with 3 (3×3, 3×4, 3×5, and 3×6), three combinations with 4 (4×4, 4×5, and 4×6), two combinations with 5 (5×5 and 5×6), and one combination with 6 (6×6). Because the board has thirty-six (6×6) cells, each product can appear three times and six products can appear more than three times. We usually use the more difficult products for the remaining cells, such as 36, 36, 30, 30, 25, and 24. (We omitted the combinations 4×3, 5×3, 5×4,

Figure 2

Four-in-a-Row boards

24	9	20	15	30	18
12	30	25	36	24	16
36	15	9	18	20	36
16	36	30	25	12	30
12	20	25	15	24	36
24	16	30	9	25	18

3 4 5 6

(a) A Four-in-a-Row board with factors 3–6

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

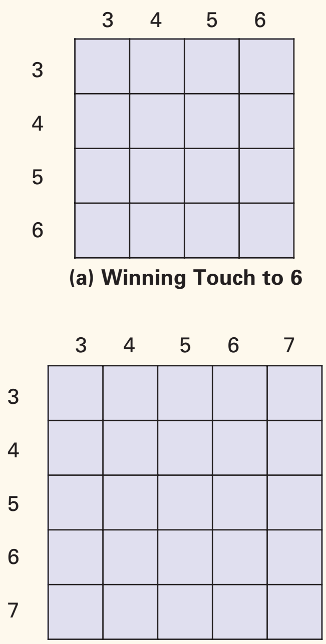
1 2 3 4 5 6 7 8 9

(b) A common Four-in-a-Row board

Children run through a practice game before entering into competition.

Figure 3

Two boards for Winning Touch



(a) Winning Touch to 6

(b) Winning Touch to 7

6 × 3, 6 × 4, and 6 × 5 from this consideration because 4 × 3, for example, was the same problem as 3 × 4 to our students.)

Winning Touch

Figure 3a shows the board for Winning Touch to 6 and **figure 3b** shows the board for Winning Touch to 7. These boards are modifications of a commercially made game called The Winning Touch (Educational Fun Games 1962). This ready-made game involves all the factors from 1 to 12 and uses a much larger (12 × 12) board than the boards in **figure 3.** A chart on the inside of the cover shows all one hundred forty-four products, and the instructions in the box advise the players to consult this chart when they are unsure of a product.

We took the chart out of the game because it motivates children not to learn products. When children can look up a product quickly, they are deprived of an opportunity to learn it through the exchange of viewpoints among the players. The second modification we made was to eliminate factors less than 3 and reduce the range of factors. For example, when we made the board for factors from 3 to 6, we called it Winning Touch to 6 (see **fig. 3a**). As the class became ready to move on to

more difficult factors, we made new boards and called them Winning Touch to 7 (see **fig. 3b**), and so on. We eliminated factors greater than 10, as well as 10, from the game.

Two or three people can play this game. Winning Touch to 6 uses sixteen tiles, on which are written the sixteen products (9, 12, 15, and so on) corresponding to the columns and rows. All the tiles are turned facedown and mixed well, and each player takes two tiles to begin the game. The players look at their two tiles without letting anyone else see them.

The first player chooses one of his or her tiles and places it in the square corresponding to the two factors. For example, 25 must be placed in the column labeled "5" that intersects the row labeled "5." The first player then takes one tile from the facedown pile to have two tiles again. The players take turns placing one tile at a time on the board. To be played, a tile must share a complete side with a tile that is already on the board. Touching a corner is not enough. For example, if the first player has played the tile marked 25, the only products that the second player can use are 20 and 30.

If a player does not have a tile that can be played, he or she must miss a turn, take a tile from the facedown pile, and keep it in his or her collection. In other words, the player cannot play this tile during this turn. The person who plays all his or her tiles first is the winner. If a player puts a tile on an inappropriate square, the person who catches the error can take that turn, and the person who made the error must take the tile back.

When the students are fairly certain about most of the products, it is time to work for mastery and speed. The next section discusses Around the World, Multiplication War, and Arithmetiles.

Games Requiring Speed

Around the World

In this whole-class activity, the teacher shows a flash card and two children at a time compete to see who can give the product of two numbers faster. To begin, the whole class is seated except for the first child, who stands behind the second child to compete. The winner stands behind the third child, and these two wait for the teacher to show the next flash card. The child who wins stands behind the fourth child, and so on, until everyone has had a chance to compete. If the seated child beats the standing child, the two exchange places, and the winner moves to the next person. A child who defeats many others and makes it to the end by moving from classmate to classmate is the champion who has gone "around the world."

Some teachers feel that Around the World benefits only students who already know most of the multiplication facts. When used skillfully, however, this game can motivate students to learn more combinations at home.

Multiplication War

War is a simple game that uses regular playing cards. In the traditional game, the cards are first dealt to two players, who keep them in a stack, facedown, without looking at them. The two players simultaneously turn over the top cards of their respective stacks, and the player who has the greater number takes both cards. The winner of the game is the person who wins the most cards.

Multiplication War is a modification of War. We begin by using cards up to 5 and later add the 6s, 7s, 8s, and 9s gradually. After dealing the cards, the two players simultaneously turn over the top cards of their respective stacks, and the person who announces the correct product first wins both cards. The winner of the game is the player who collects the most cards. It is up to the two players to decide, before beginning the game, what happens in case of a tie.

Arithmetiles

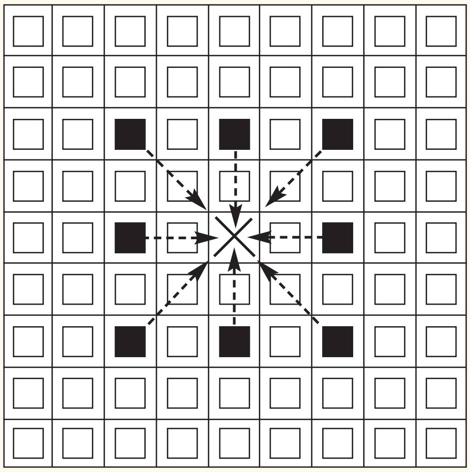
This is a modified version of a commercially made game called Arithmechips (Lang 1990). Arithmechips uses a board that has a grid of eighty-one (9×9) squares and one hundred fifty-six chips. Most of the chips have a multiplication problem on one side and the corresponding product on the other side. To begin the game, eighty chips are randomly placed in every square of the board except the one in the middle marked "X," with the problem side up. The players win chips by jumping over one chip at a time, as in Checkers, reading aloud the problem on the chip they just jumped, stating the answer, and turning the chip over to verify the answer. If the answer is correct, the player can keep that chip.

We modified this game and called it "Arithmetiles." We made the following modifications:

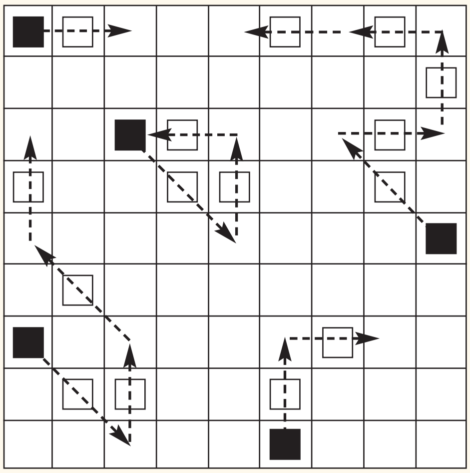
- Eliminating factors of 0, 1, 11, and 12
- Introducing the requirement of speed
- Eliminating the possibility of "self-correction" by not writing a product on each chip

Figure 4

Possible jumps in Arithmetiles



(a) The eight possible jumps at the beginning of the game



(b) Possible moves involving one or more jumps

- Eliminating the requirement of having to read the problem aloud before stating a product
- Introducing levels of difficulty

Arithmetiles is a three-player game played with a 9 × 9 grid that has an "X" in the middle. The game requires eighty problems because players must fill all the squares in the grid except one with tiles that have multiplication problems such as 6 × 7 on them. But because there are only sixty-four combinations of the factors 2–9, sixteen problems must appear on more than one tile. We use the following more difficult combinations on the sixteen tiles: 6 × 6, 6 × 7, 6 × 8, 6 × 9, 7 × 6, 7 × 7, 7 × 8, 7 × 9, 8 × 6, 8 × 7, 8 × 8, 8 × 9, 9 × 6, 9 × 7, 9 × 8, and 9 × 9.

The eighty tiles are placed, facedown, on all the squares except the one marked "X." The first player may play any one of the tiles marked in black in **figure 4a** and jump over a tile into the empty cell marked "X," vertically, horizontally, or diagonally. He or she quickly turns over the jumped tile and announces the product. If the other two players agree with the product and the speed with which the player announced it, the first player can keep the jumped tile. If the product is incorrect, the person who was first to correct it can keep the tile in question. If the other two players agree that the first player gave the answer too slowly, the jumped tile is returned to the grid and the turn passes to the next player.

The X cell is filled after the first play. The second player can choose any tile that he or she wishes to jump vertically, horizontally, or diagonally into the vacated cell. Play continues in this manner, as in Checkers. The person who collects the most tiles is the winner.

As **figure 4b** shows, making two or more jumps is possible. To make multiple jumps, a player must keep his or her hand on the tile while stating the first product and every subsequent product.

Teachers can make Arithmetiles more difficult by eliminating the sixteen easy products of 2–5 that appear in **figure 1.** In this version, we are left with only 64 – 16 = 48 combinations of factors. To have eighty problems, players must use most combinations twice and some combinations only once.

How We Used the Games

Motivation to learn the multiplication tables must come from within the child. The teacher has much to do with the development of this motivation, however. Toward the end of the year, our students' desire to beat the teacher in Multiplication War and Arithmetiles inspired them to learn the tables. A similar motivation was to beat the "stars" in the class. When many students knew the tables rather well, the teacher began to challenge as many groups as possible every day. She briefly played with one group, left the students to continue playing by themselves, and went on to the next group, asking, "Who's going to beat me today?" Some

students made flash cards to practice at home, and a few were observed quizzing each other with flash cards on the bus during a field trip.

The children were motivated to learn the multiplication combinations because the games were fun and had a lot of variety. There was no coercion, timed tests, or the threat of a bad grade. Of course, the teacher explained how this knowledge would help in fourth grade, but students largely ignored such talk about next year. When the teacher played every day with small groups of children, they received a stronger message: that games are important enough for the teacher to play.

What about the games was fun to a third grader? Students made decisions every day about which game to play and with whom. Deciding whom to play with was especially a "big deal." Students who had mastered many of the combinations wanted to play against someone at the same level. Those who were not fluent wanted to play against someone at their level so that they still had a chance of winning. A difficult game such as Arithmetiles was not popular with the slower students. They tended to choose games such as Winning Touch, which did not penalize them for lack of speed.

The teacher's role was considerable in giving choices and maximizing learning. We deliberately introduced the more difficult factors one at a time. For example, when we introduced 6 as a multiplier, we played Winning Touch to 6, Four-in-a-Row to 6, Multiplication War with cards only to 6, and Salute, also with cards to 6. We played these games over a two-week period using factors up to 6. After that, we focused on factors up to 7 for about a week, then factors up to 8 and 9.

After a month, when the students had played all these games at four different levels of difficulty, the teacher began to announce on some days that everyone had to play a game with sevens or that everyone had to play Winning Touch at their "just right" level. She also introduced other games such as PrimePak (Conceptual Math Media 2000) and Tribulations (Kamii 1994). The children also benefited from whole-class discussions of strategies. In one of the discussions, for example, one child said that multiplying any number by 8 is easy if "you double it and double it and double it," meaning that 8×6 can be done easily by doing $2 \times 6 = 12$, $2 \times 12 = 24$, and $2 \times 24 = 48$.

As the year progressed, the students selected appropriate partners and games. Some stuck with the same game for a long time; they needed time to develop comfort with certain combinations. Everyone learned the multiplication combinations and enjoyed doing so.

References

Carpenter, Thomas P., Ellen Ansell, Megan L. Franke, Elizabeth Fennema, and Linda Weisbeck. "Models of Problem Solving: A Study of Kindergarten Children's Problem-Solving Processes." *Journal for Research in Mathematics Education* 24 (November 1993): 428–41.

Clark, Faye B., and Constance Kamii. "Identification of Multiplicative Thinking in Children in Grades 1–5." *Journal for Research in Mathematics Education* 27 (January 1996): 41–51.

Conceptual Math Media. *PrimePak.* San Francisco: Conceptual Math Media, 2000.

Educational Fun Games. *The Winning Touch.* Winnetka, Ill.: Educational Fun Games, 1962.

Kamii, Constance. *Young Children Continue to Reinvent Arithmetic, 3rd Grade.* New York: Teachers College Press, 1994.

———. *Young Children Reinvent Arithmetic, 2nd ed.* New York: Teachers College Press, 2000.

Lang, Audrey Clifford. *Arithmechips.* Creative Toys Ltd., 1990.